2   The Tonal Properties of Pitch-Class Sets: Tonal Implication, Tonal Ambiguity, and Tonalness

David Temperley

Eastman School of Music
26 Gibbs St.
Rochester, NY 14604
dtemperley@esm.rochester.edu

Abstract

This paper explores three tonal properties of pitch-class sets. *Tonal implication* is the key that a set implies; *tonal ambiguity* is the degree to which a set implies one key as opposed to several; *tonalness* is the degree to which a set is characteristic of the language of common-practice tonal music. I propose a simple probabilistic framework for measuring these three properties. I then examine some applications of the theory, focusing on the measurement of tonal ambiguity and how this sheds light on some issues of compositional practice and tonal analysis.

2.1   Introduction

The concept of a pitch-class set is a familiar one in music theory. In the study of early 20th-century atonal music, it has proven to be a concept of great theoretical and analytical value. Its relevance to tonal music is, perhaps, less obvious. In fact, however, references to pitch-class sets are rather commonplace in discourse on tonal music, though the term itself is rarely used.[1] For instance, when we speak of a "C major triad," we generally have in mind a set consisting of the pitch-classes C, E, and G, so that—for example—the three passages in Figure 2.1 are all realizations of the set.
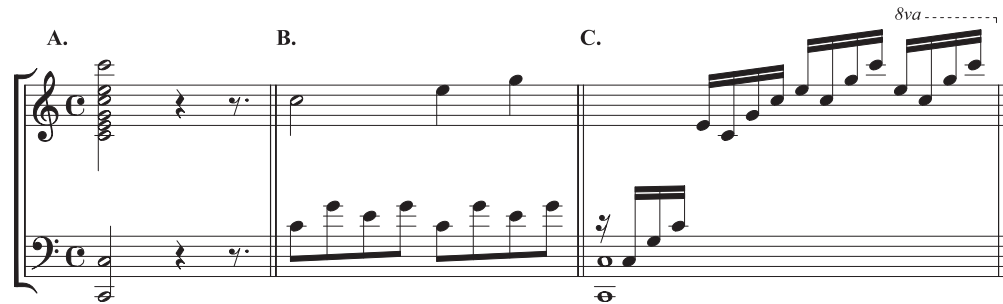

Figure 2.1. *Three realizations of a C major triad.* **(a)** *Mozart, Piano Concerto K. 503, I, m. 1;* **(b)** *Mozart, Sonata K. 545, I, m.1;* **(c)** *Chopin, Etude Op. 10 No. 1, m. 1.*

The premise of this article is that pitch-class sets have certain properties that are an important part of the way we perceive and understand tonal music. We will be concerned with three properties in particular: tonal implication, tonal ambiguity, and what I will call "tonalness." These three properties are illustrated by the three sets in Figure 2.2.

Figure 2.2. *Three pitch-class sets.*

Tonal implication is simply the tonality or the key that a set implies. For example, the set [C-D-E-F] seems to imply the key of C major, or perhaps F major; it certainly does not imply F# major. The set [B-C-D-Eb] seems to imply C minor, while a set like [C-C#-F#-G] does not project a strong implication of any key.

Tonal ambiguity is closely related to tonal implication, and is the extent to which a set strongly implies a single key or is ambiguous among multiple keys. [B-C-D-Eb] is quite unambiguous; it has a strong implication of C minor. [C-D-E-F] is more ambiguous because it could easily imply either C major or F major. [C-C#-F#-G] is highly ambiguous too, as (again) it does not strongly imply any key.

Finally, tonalness is the degree to which a set seems characteristic of the language of common-practice tonality. Tonalness asks the question: Is this a set that we would expect to find as the pitch material for a short segment of a tonal piece? It seems safe to say that [C-D-E-F] and [B-C-D-Eb] are both fairly high in tonalness, whereas [C-C#-F#-G] is lower in tonalness. In what follows, I propose a probabilistic method for predicting these tonal properties of pitch-class sets.

Before continuing, we should acknowledge the obvious fact that the tonal properties of a pitch-class set depend in some measure on the way the set is realized—the way the pitch-classes are doubled, repeated, and arranged in time and register. One could certainly realize the set [C-D-E-F] in a way that unambiguously conveyed C major (Figure 2.3a) or that unambiguously conveyed F major (Figure 2.3b).[2]

Figure 2.3. *Two realizations of the set [C-D-E-F].*

Yet we do, I submit, have intuitions about the tonal properties of pitch-class sets—intuitions to which I was appealing above.

What is the basis for these intuitions? There are various ways that the notion of "the properties of a pitch-class set" could be defined more rigorously. One could define a set's properties as those produced by a "neutral" realization of the set—perhaps a repeating monophonic pattern, fading in and out, in which each tone is presented once, and all are durationally equal and realized as Shepard tones. One could also define the properties of a set in terms of the properties of all of its possible realizations (given some kind of defined limits on the space of possible realizations). For example, the tonal ambiguity of a set could be defined as the average ambiguity of all possible realizations of the set (or, what comes to the same thing, as the average ambiguity of a large number of randomly chosen realizations). Both of these approaches essentially identify a pitch-class set with some kind of prototypical or neutral realization (or realizations) of that set; that is also what we are doing, I believe, in making intuitive judgments about the set. We will not pursue this issue further here. Suffice to say that the notion of "the properties of a set," while somewhat vaguely defined, is certainly not without empirical content and could be specified more precisely if one desired to do so.

2.2 Tonal Implication, Tonal Ambiguity, and Tonalness

2.2.1 Tonal Implication

We begin with the concept of tonal implication. Probabilistically, the tonal implication of a set can be defined in terms of the probability of each key given the set $P(\text{key} \mid \text{pcset})$; the set yielding the highest probability is the most strongly implied key. Bayes' rule tells us that

$$(1) \qquad P(\text{key} \mid \text{pcset}) \propto P(\text{pcset} \mid \text{key})\, P(\text{key})$$

when the pcset is held constant. Let us assume that all keys are equal in probability. (This seems a reasonable assumption for listeners who lack absolute pitch, as most listeners do.) Then it follows that $P(\text{key} \mid \text{pcset}) \propto P(\text{pcset} \mid \text{key})$. Thus the probability of a key given a pitch-class set is proportional to the probability of the pitch-class set given the key.

How can we determine the probability of a pitch-class set given a key? Elsewhere I have proposed a way of addressing this problem (Temperley 2004). This method requires a musical corpus; in this case, the corpus we use is the Kostka-Payne corpus, a set of 46 harmonically analyzed excerpts from tonal pieces taken from the Kostka-Payne theory workbook (Kostka and Payne 1995). The corpus is divided into short segments, corresponding roughly to measures (see Temperley 2004 for further detail on how this was done). For all major-key segments, we count the proportion of segments containing each "scale-degree"—a pitch-class in relation to the key; we repeat the same process for minor keys. This yields the "key-profiles" shown in Figure 2.4. (The concept of key-profiles is familiar from the work of Krumhansl 1990. In this case, however, we construe key-profiles probabilistically, which Krumhansl did not.)

It can be seen, for example, that in major-key segments, scale degree 1 occurs in .748 (74.8%) of segments; scale degree #4 occurs in only .096 (9.6%) of segments. The key-profiles reflect well-established musical principles; tones of the scale have higher values than chromatic tones (assuming the minor scale to be harmonic minor), and tonic-triad tones have higher value than other scalar tones. Given these profiles, the probability of a pitch-class set given a key can be defined as

$$(2) \qquad P(\text{pcset} \mid \text{key}) = \prod_{p} K_{pc} \prod_{\sim p} (1 - K_{pc})$$

where K_{pc} is the the key-profile value for a pitch-class given a key, p indicates pitch-classes present in the set, and $\sim p$ indicates pitch-classes not present in the set. As noted above, this is proportional to $P(\text{key} \mid \text{pcset})$; thus the key that maximizes this expression is the most probable key given the set. A key-finding algorithm based on this approach has proven to be quite successful; on the Kostka-Payne corpus, the model yielded the correct key in 775 out of 896 segments (86.5%). (For further details about this and other tests, and comparison of this model with other key-finding models, see Temperley 2007.)
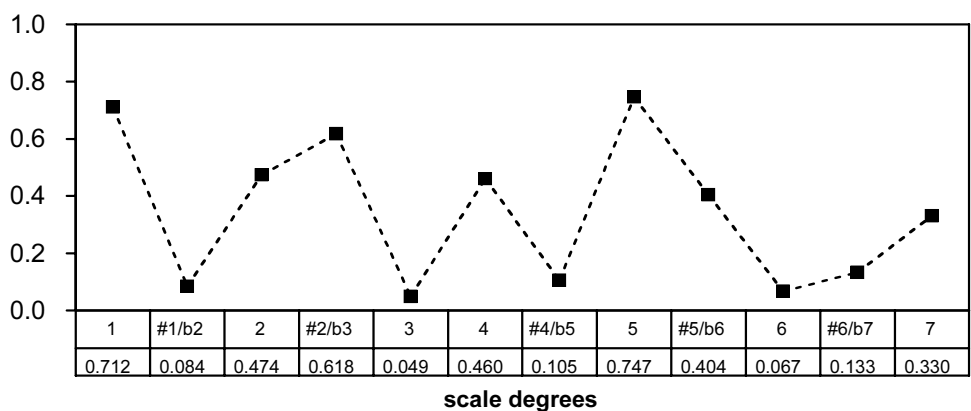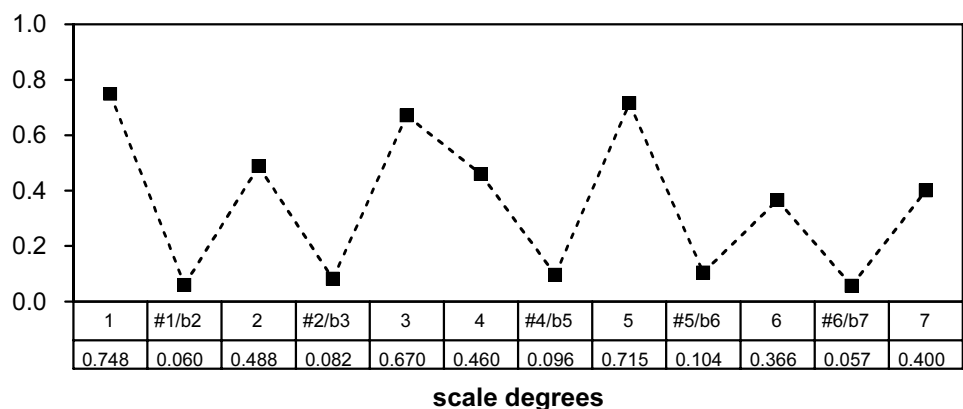
1	#1/b2	2	#2/b3	3	4	#4/b5	5	#5/b6	6	#6/b7	7
0.748	0.060	0.488	0.082	0.670	0.460	0.096	0.715	0.104	0.366	0.057	0.400

scale degrees

1	#1/b2	2	#2/b3	3	4	#4/b5	5	#5/b6	6	#6/b7	7
0.712	0.084	0.474	0.618	0.049	0.460	0.105	0.747	0.404	0.067	0.133	0.330

scale degrees

Figure 2.4. *Key-profiles for major keys (above) and minor keys (below).*

2.1.2   Tonal Ambiguity

We now turn to ambiguity. As noted above, the ambiguity of a set is simply the de-gree to which it clearly implies a single key, or is equivocal between two or several keys. It seems clear that the key-finding method described above could also be used to measure ambiguity. A set for which one key is far more probable than any other is unambiguous; a set that yields roughly equal probabilities for multiple keys is am-biguous. There are various ways that this might be quantified. One simple way is to define the ambiguity of a set as the ratio in probability between the most probable key and the next most probable; if this value is close to 1 the set is ambiguous (it can never be *less* than 1), while if it is much greater than 1 the set is unambiguous. It is important to note that a *higher* "top-two ratio" indicates *lower* ambiguity; rather than describe this as an indicator of tonal ambiguity, it would be more precise to call it an indicator of *tonal clarity*, and I will henceforth do so here.

Table 2.1 shows this measure of tonal clarity for certain basic pitch-class sets. The first two columns show the most probable and second-most probable keys, with their

probabilities; the third column is the ratio between the two, yielding the "tonal clarity score" for the set. We can see, for example, that [C-E-G] is judged to have higher clarity (lower ambiguity) than [C-C#-D] by this measure, as it surely should. Turning to larger pitch-class collections, we find that the octatonic and whole-tone scales are very low in clarity, close to or at the minimum score of 1; the same is true of the set [C-C#-D-Eb-E-F-F#] (seven pitch-classes spaced by half-steps, which we call by its pitch-class set-theory name, [0123456]). (In fact, an octatonic scale has four keys that are tied for top rank, and a whole-tone scale has six; it is arbitrary in such cases to decide which of the keys are first- and second-ranked, but whichever ones are, the ratio between their probabilities is clearly 1.) The major scale has much higher clarity, and the harmonic minor scale has the highest of all.

In some respects, these results are unsurprising. It is common knowledge that the whole-tone scale and octatonic scale are highly ambiguous: they are symmetrical scales, formed by a repeating intervallic pattern, thus they yield the same degrees in several different keys. For example, the whole-tone scale [C-D-E-F#-Ab-Bb] contains 1-2-3-#4-b6-b7 of C major, and the same scale-degrees in D major, E major, F# major, Ab major, and Bb major. By contrast, the major and harmonic minor scales are not symmetrical: each transposition of the major scale contains the scale-degrees of only one major key. One aspect of Table 1 that is surprising, and *not* part of conventional wisdom, is the greater clarity of the harmonic minor scale compared to the major scale. Closer examination proved that this was due to two factors. One concerns the degree of overlap between scales of different keys. A major scale such as C major shares 6 out of 7 degrees with three other scales (G major, F major, and A minor). By contrast, a minor key such as A minor (represented by the harmonic minor scale) shares 6 out of 7 scale-degrees with just one key, C major. There is another factor as well, relating specifically to the relationship between relative keys. In the case of C major, the three tonic-triad notes of all three "nearest-neighbor" keys (G major, F major, and A minor) are all within the scale; this means these keys have reasonably high probability given the C major scale. In the case of A minor, however, C major—the only nearest-neighbor key—has only *two* tonic-triad notes within the A minor scale. Together, these two factors explain why a major scale is judged by the model as more ambiguous than a harmonic minor one. I will argue that this difference between major and minor scales has interesting musical consequences.

2.2.3 Tonalness

Finally, we consider tonalness. I defined tonalness earlier as "the degree to which a set seems characteristic of the language of common-practice tonality." In probabilistic terms, we can think of this as the *overall* probability of a set occurring in a short segment of a tonal piece—$P(\text{pcset})$. By basic rules of probability,

$$(3) \qquad P(\text{pcset}) = \sum_{\text{key}} P(\text{pcset} \cap \text{key}) = \sum_{\text{key}} P(\text{pcset} \mid \text{key})\, P(\text{key})$$

We already derived an expression for P(pcset|key) [see equation (1) above]. If we assume that each key has an equal probability of 1/24, we can easily calculate the expression above. This gives us a measure of the probability of occurrence of a pitch-class set.

Table 2.2 shows tonalness values for some of the same sets considered earlier with regard to ambiguity. The tonalness value itself—the overall probability of the set—is shown in the left-hand column. It can be seen that, among three-pc sets, the major and minor triad have higher probability than the diminished triad, which in turn has higher probability than [C-C#-D]. No doubt this is because the major and minor triads are highly probable in combination with certain keys, namely keys in which they are diatonic triads (and the tonic triad in particular). By contrast, the set [C-C#-D] is not particularly probable given *any* key, as it will always contain at least one chromatic note. This is made clear by the second column, which shows the joint probability of each set with its most probable key. The table also shows the probabilities of several larger pitch-class collections. It can be seen that the major scale is somewhat more probable than the harmonic minor scale; both are much more probable than the whole-tone scale, the octatonic scale, and [0123456].[3]

An interesting issue that arises here is the relationship between clarity and tonalness. Table 2.2 gives both tonalness and clarity ratings for the same sets, so that these values can be compared. One might assume, at first thought, that clarity and tonalness usually go together. Famously ambiguous (low-clarity) sets such as the octatonic scale and whole-tone scale are low in tonalness; highly tonal scale collections such as the major and harmonic minor scales are high in clarity. However, consider the set [C-D-E-G-A-B]—sometimes known as the diatonic hexachord. This set is very low in clarity, but it is rather high in tonalness; its tonalness value is much higher than that of the whole-tone scale, which also has six notes. The high tonalness of this set is due to the fact that it is highly probable given two different keys (C major and G major), but this also accounts for its low clarity. Could there also be pitch-class sets which are low in tonalness, but high in clarity? This is more difficult to imagine; if a set is low in tonalness, it is unlikely to be contained in any major or harmonic minor scale, but any such set would probably tend to be low in clarity as well.

2.3   Applications

In the first half of this article, I presented ways of quantifying the tonal implication, tonal ambiguity, and tonalness of a pitch-class set. I submit that these measures accord rather well with our intuitions about the tonal properties of different sets. There are a number of applications of this theory that might be considered. I would like to focus here on the measurement of tonal ambiguity, and how this can shed light on several issues of compositional practice and tonal analysis.

	Best key (with P(key | pcset))		Second-best key (with P(key | pcset))		Tonal clarity score
Chords					
C-E-G (C major triad)	C	(.597)	Em	(.130)	4.60
C-Eb-G (C minor triad)	Cm	(.550)	Eb	(.134)	4.11
C-Eb-Gb (C dim. triad)	Bbm	(.136)	Cm	(.122)	1.11
C-C#-D ([012])	Gm	(.112)	F#m	(.089)	1.26
Larger sets					
C-D-E-F-G-A-B (C major scale)	C	(.658)	Am	(.159)	4.14
C-D-Eb-F-G-Ab-B (C harmonic minor scale)	Cm	(.908)	Eb	(.054)	16.78
C-D-E-F#-Ab-Bb (Whole-tone scale)	Gm	(.100)	C#m	(.100)	1.00
C-D-Eb-F-Gb-Ab-A-B (Octatonic scale)	Ebm	(.190)	Cm	(.190)	1.00
C-C#-D-Eb-E-F-F# ([0123456])	Db	(.218)	Bbm	(.214)	1.02
Other sets					
C-Eb-F#-A	Em	(.192)	C#m	(.192)	1.00
C-D-E-G-B	C	(.481)	G	(.373)	1.29
C-D-E-G-A-B	G	(.438)	C	(.342)	1.28
C-D-Eb-F-G-Ab	Cm	(.713)	Eb	(.180)	3.95
D-Eb-F-G-Ab	Eb	(.460)	Cm	(.425)	1.08
Cadence sets					
V-I (C-D-E-G-B)	C	(.481)	G	(.373)	1.29
V7-I (C-D-E-F-G-B)	C	(.832)	Am	(.047)	17.71
V-i (C-D-Eb-G-B)	Cm	(.684)	G	(.161)	4.25
V7-i (C-D-Eb-F-G-B)	Cm	(.842)	C	(.056)	15.03

Table 2.1. *Tonal clarity values for some common pitch-class sets.*

Pitch-class set	Overall probability of set ("tonalness")	Joint probability of set with most probable key		Tonal clarity score
C-E-G (major triad)	.00173	.00103	(C)	4.60
C-Eb-G (minor triad)	.00178	.00098	(Cm)	4.11
C-Eb-Gb (dim. triad)	.00032	.00004	(Bbm)	1.11
C-C#-D ([012])	.00022	.00002	(Gm)	1.26
C-D-E-F-G-A-B (major scale)	.00049	.00032	(C)	4.14
C-D-Eb-F-G-Ab-B (harmonic minor scale)	.00028	.00025	(Cm)	16.78
C-D-E-F#-Ab-Bb (whole-tone scale)	.00001	.000001	(Gm*)	1.00
C-D-Eb-F-Gb-Ab-A-B (octatonic scale)	.000004	.0000007	(Ebm*)	1.00
C-C#-D-Eb-E-F-F# ([0123456])	.000006	.000001	(Db)	1.02
C-D-E-G-A-B (diatonic hexachord)	.00111	.00049	(G)	1.28

Table 2.2. *Tonalness of some common pitch-class sets. Symmetrical sets (such as the whole-tone scale, octatonic scale, and aggregate) yield equal probability judgments for multiple keys; in this case the model makes an arbitrary decision.*

2.3.1 Cadential Usage

The first issue concerns the use of cadences. It is well known that most tonal pieces (and large sections of pieces) end with a "perfect authentic cadence": a root-position V chord followed by a root-position I, with the tonic pitch in the melody of the I chord. This is almost required at the end of a piece—at least, if the composer intends to create a sense of full closure in the tonic key. The key of the piece may be either major or minor, and the V chord may be either a V triad or a V7 chord; this creates four possibilities, as shown in Figure 2.5.

Figure 2.5. *Perfect cadences: (a) V-I in major, (b) V7-I in major, (c) V-i in minor, (d) V7-i in minor.*

The four progressions shown in the figure yield four different pitch-class sets, with differing degrees of ambiguity (see the final section of Table 1, "cadence sets"). Both progressions involving V7 are extremely high in clarity, as is the V–i progression in minor, employing [C-D-Eb-G-B]. The V–I progression in major, however, employs the set [C-D-E-G-B]. This set is fully contained in two major scales, C major and G major (and no harmonic minor scales); moreover, it contains the tonic triads of both keys. Since the set is equally compatible with the two keys at both scalar and triadic levels, we would expect it to be highly ambiguous; the model confirms this, assigning a clarity value of just 1.29. This suggests that the V–I progression should seem less decisive than the V–i progression in its key implications. To my ears, this is indeed the case. Admittedly, a progression like that in Figure 5a (even out of context) implies C major more strongly than G major (probably because this two-chord gesture has strong cadential associations), but it lacks the unshakable confidence and certainty of Figure 5c. (This point relates to our earlier discussion of the major and harmonic minor scales, where we observed that the harmonic minor is higher in clarity than the major scale.)

These observations suggest a prediction about compositional practice. At the final cadence of a piece, we can assume that composers (at least in the common-practice period) generally sought to achieve a maximal sense of tonal stability and closure. Thus we might expect, in final cadences of major-key pieces, that composers would tend to favor V7–I over V–I cadences, given the rather low clarity of the V–I variant. In minor-key pieces, by contrast, both the V7–i and V–i variants of the cadence are

very high in clarity; thus the current theory does not predict any strong preference for the V7–i variant.

This prediction was tested with regard to three corpora: the piano sonatas of Haydn, Mozart, and Beethoven. All movements in all sonatas of all three composers (with the exception of Haydn's early piano sonatas) were examined to see if the key was major or minor and if the final cadence contained a V or V7 chord. Table 2.3 shows the results. For all three composers, it can be seen that the proportion of V–I cadences in minor-key movements is much higher than in major-key movements; in all three corpora, this difference is statistically significant. Thus, for the classical-period composers at least, there does indeed seem to be a stronger preference for V7 over V with major-key pieces, just as the current model predicts.

	Haydn		**Mozart**		**Beethoven**	
	Major	Minor	Major	Minor	Major	Minor
V	2 (12.5%)	4 (80.0%)	5 (23.8%)	2 (66.7%)	4 (15.4%)	8 (72.7%)
V7	14 (87.5%)	1 (20.0%)	16 (76.2%)	1 (33.3%)	22 (83.6%)	3 (27.2%)
Total	16	5	21	3	26	11
	$\chi^2 = 8.5$, p<.01)		$\chi^2 = 2.3$, p<.13)		$\chi^2 = 11.6$, p<.001)	

Table 2.3. *Final cadences in Haydn, Mozart, and Beethoven piano sonatas. This table represents information about the final cadences of movements in piano sonatas by Haydn, Mozart, and Beethoven. All Haydn sonatas from Hob. XIV:27 onward were examined, as well as all Mozart and Beethoven sonatas. "Major" or "minor" indicates the key of the movement; "V" or "V7" indicates whether the dominant of the final cadence is a V or V7. An objective algorithm, which will not be described here, was used to identify the final cadence of the piece. A number of movements were eliminated from consideration because they had no clear final cadence according to this algorithm.*

2.3.2 Ambiguous Sets in Common-Practice Pieces

Here I consider some highly ambiguous pitch-class sets and some interesting uses of these sets in common-practice pieces. As noted earlier, it is well known that certain scalar collections are highly ambiguous, such as the whole-tone and octatonic scales. Also well known is the ambiguity of certain tonal chords, such as the diminished seventh chord. This, too, is a symmetrical pitch-class set; the chord [C-Eb-F#-A] contains the scale degrees 7-2-4-6 in E minor, and contains the same degrees in C# minor, G minor, and Bb minor. Naturally, then, the model assigns it the minimal clarity score of 1.0 (see Table 2.1). A passage built on a diminished seventh chord gives an effect of extreme tonal instability and disorientation, as in the development section of Beethoven's *Appassionata* sonata. Common-practice composers sometimes also employ ambiguous pitch-class sets in more subtle ways. Consider the set [C-D-E-G-B]—

a highly ambiguous set, as discussed above. A very simple use of this set, one which seems to maximize its tonal ambiguity, is found in the opening of Chopin's Mazurka Op. 24 No. 2 (Figure 2.6). The passage alternates between a C major triad and a G major triad, both in root position—thus even harmonic factors cannot really tip the balance to one key or the other. (Even the metrical placement of the chords seems deliberately evenhanded: the "cross-rhythm" pattern places C major on the downbeat in mm. 1 and 3, and G major in mm. 2 and 4.) The progression begins on a C major chord, perhaps slightly favoring C major—but one might just as well say that ending on a G major chord favors that key. The piece proves to be in C major, in fact, but one could just as easily imagine it continuing in G major.
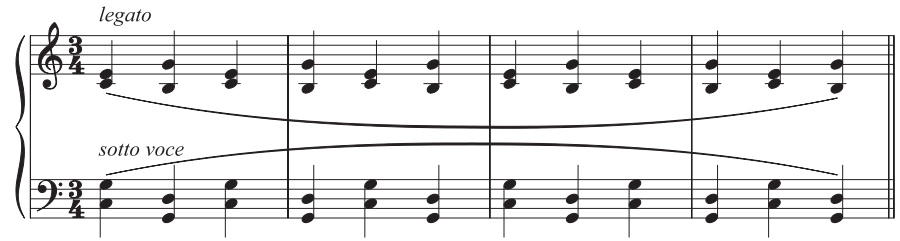

Figure 2.6. *Chopin, Mazurka Op. 24, No. 2, mm. 1–4.*

If one adds an A to the set discussed above, this produces [C-D-E-G-A-B], sometimes known as the "diatonic hexachord." As discussed earlier, this set too is highly ambiguous, as it occurs in two major scales (C major and G major) and contains both complete tonic triads; its clarity value is just 1.28. An example of this set (transposed to [G-A-B-D-E-F#], implying G major or D major) is seen in the first two measures of the aria of Bach's Goldberg Variations (Figure 2.7).

Figure 2.7. *Bach, Goldberg Variations (BWV 988), Aria, mm. 1–8.*

As in the Chopin mazurka, the harmony is also equivocal between these two keys: G major in m. 1 followed by D major in m. 2, with the root featured prominently in the melody in both cases. The following two measures introduce C#, tonicizing D major; but the following four measures (mm. 5-8) replace C# with C, and establish G major beyond all doubt. One might question whether mm. 1–2 are really ambiguous. Of course, we know (if we know the piece) that it is in G major, and that may bias us toward a G major interpretation over a D major one. I would argue, however, that mm. 1-2 do contain a subtle element of ambiguity. A I–V6 progression, though perhaps favoring the first chord as tonic because of its primacy, can also be interpreted as IV–I6 (favoring the second chord as tonic) without difficulty. Admittedly, the effect of the ambiguity in the Bach is very different from that of the *Appassionata* development section. The feeling here is not one of complete disorientation, as in that case, but rather a subtle undercurrent of indecision or equivocation between two clear alternatives, followed by a leaning toward one (D major in mm. 3–4) and then a much stronger commitment to the other (G major in mm. 5–8).

Several other pitch collections deserve consideration with regard to ambiguity. One is [C-D-Eb-F-G-Ab]—sometimes called a minor hexachord. In scalar terms, this set is ambiguous between two "relative" keys, C minor and Eb major in this case. However, there is an important difference between this and the diatonic hexachord; in this case, the C minor tonic triad is fully included in the set, while the Eb major triad is not. Thus we would expect the model to favor C minor quite strongly, and it does, assigning a clarity value of 3.95. This seems true to perception. Even in cases where the set is presented in such a way that it does not strongly outline a particular harmony, the minor key seems to prevail over the major one. The opening of Schoenberg's *Verklärte Nacht* is one example, where a downward scalar presentation of the hexachord ([Bb-A-G-F-E-D]) seems to favor D minor over F major (Figure 2.8). Of course, this is reinforced by the D pedal tone; but even disregarding this, the D minor implication seems to prevail. (Another factor in this case is that the D in the descending hexachord is not followed closely by step; this tends to favor it as a chord-tone, thus implying D minor harmonically. If we imagine the pattern in reverse, [D-E-F-G-A-Bb], the leaning towards D minor is somewhat less strong, but still present, in my opinion.)
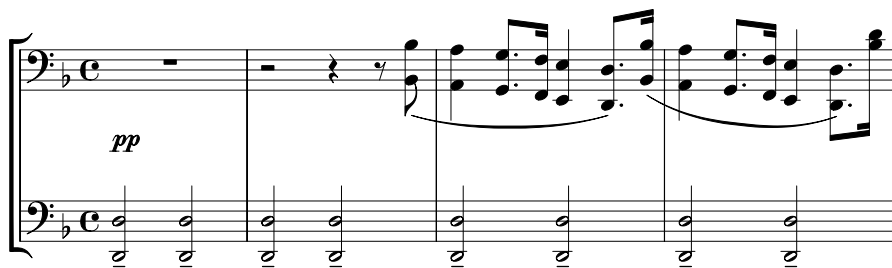

Figure 2.8. *Schoenberg, "Verklärte Nacht," mm. 1–4.*

A more ambiguous case is the set [D-Eb-F-G-Ab], the same as the minor hexachord except with the C deleted. This set is still ambiguous between the same two keys (C minor and Eb major) in scalar terms, but now it has exactly two notes from the tonic triad of each (Eb and G from C minor, and Eb and G from Eb major)—though the fact that it contains the tonic of Eb major and not of C minor may slightly favor the former. We would therefore expect the set to be quite evenly balanced between these two keys, and the model agrees, assigning a clarity score of 1.08. The ambiguity of this set is sometimes exploited quite effectively; a well-known example is the opening of Brahms's Clarinet Quintet, using [C#-D-E-F#-G] (implying D major or B minor) (Figure 2.9).

Figure 2.9. *Brahms, Clarinet Quintet, first movement, mm. 1–5.*

The harmony in this passage is also equivocal between the two keys, but in a different way from the examples considered earlier: in this case, the harmony *itself* is ambiguous, as the root of mm. 1 and 2 could be either B or D. Having presented these two tonalities, the piece as a whole unfolds as a series of ever larger oscillations between them: to B minor in m. 3, then to D major in mm. 4–8, then to B minor in mm. 9–28, then to D major for the second half of the exposition and finally returning to B minor in the recapitulation.

I began this paper by presenting a probabilistic model of the tonal implications of pitch-class sets and then showed how this could lead to ways of predicting the tonal ambiguity and tonalness of sets as well. I then suggested that this model yielded an interesting prediction about compositional practice—that composers would favor V7–I over V–I cadences more in major-key pieces than minor-key pieces; a study of cadence usage by classical composers confirmed this prediction. Finally, we considered some uses of highly ambiguous pitch-class sets in common-practice music, finding that composers seem to have appreciated the ambiguity of these sets and exploited it in interesting ways. It seems, then, that exploring the tonal properties of pitch-class sets is a worthwhile endeavor that can shed light on a number of issues in common-practice music.

Notes

1. An important exception is the recent body of work on "scale theory"—for example, Browne 1981, Carey and Clampitt 1989, and Clough and Douthett 1991. These authors examine basic tonal structures such as the major triad and the diatonic scale, construed as pitch-class sets. However, they focus mainly on the mathematical properties of these sets, rather than on their perceptual properties or compositional uses.

2. However, the pitch-class content of a set surely *constrains* its properties. While one could realize [C-D-E-F] to imply C major or F major, one would be hard pressed to find a realization that implied F# major, for example.

3. Notice that the overall probability figures in Table 2.2 tell us only the probability of one particular transposition of each set: for example, the C major scale. We could also consider the probability of a certain *type* of set, or "T_n set-type" (Rahn 1980)—for example, major scales in general. This would be given by the probability of one transposition of the set, multiplied by the number of distinct transpositions. In the case of the major scale, the set-type probability is .00049 × 12; in the case of the whole-tone scale, it is .00001 × 2, since there are only two distinct whole-tone scales. We will focus on particular transpositions of sets here, rather than set-types, though the two approaches usually lead to quite similar results; the major scale is far more probable than the whole-tone scale, whether one considers a single transposition of the set or all transpositions.

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