# 9 Chordal Evaluation in MIDI-Based Harmonic Analysis: Mozart, Schubert, and Brahms

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#### Abstract

New software for chordal evaluation of MIDI files with complex harmonic features represents a significant advance over earlier procedures for automated assessment used in our academy. The new software, built on generalized foundations of music-theory teaching derived from German, American, Slovak, and Czech sources, treats triads and tetrads (here called "sevenths") as collections of thirds which are either major (four half-steps) or minor (three half-steps). The software assesses cumulative durations of pitches and chord-classes after iterative evaluation of all the notes within each beat-space. Our approach to tonal theory maps easily onto MIDI key numbers. Chordal identifications are displayed underneath each system in screen notation. We have tested our routines on roughly eight dozen keyboard movements by Mozart, Schubert, and Brahms.

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## 9.1 Introduction

Harmonic analysis based on MIDI data is approached in many different ways. The origins of our project go back more than 20 years. They were originally conceptualized during the mainframe era. Our early work predated the establishment of MIDI protocols. It is rooted in harmony manuals from several sources of the later twentieth century (Piston 1946/1987, Persichetti 1961, Randel 1978, Pospíšil 1983, Kořínek 1986, Kofroň 1991, Suchoň-Filip 1992, and Filip 1997). Over the past decade we have completely rewritten our tools. We retain the terminology and concepts used earlier, but instead of encoding our own data, we now use MIDI files.

The first version of our MIDI-enabled software has been under revision since 2004. We have attempted to make chordal identification more precise and reliable. In particular we have concentrated on taking account of all note-events within each beatspace (that is, between one beat and the next) and accumulating total durations for pitches and chord-classes. Taken together, these aims require that arpeggiated notes and runs occurring after the beat be evaluated with respect to events on the previous, and sometimes the next, beat.

Some of our procedures reflect our interest in respecting the perceptual validity of harmonic material. At each beat we compare the durations of all the tones being initiated. The software then looks for tones forming thirds relations suggestive of a chord by comparing each pair of notes, starting with those of the longest values. This reflects our view that those tones which sound longer, or which are repeated more times, are also more prominent in a listener's mind, while those which are short and/or not repeated are embellishing tones. We then decide which tones belong to the chord and which are non-chordal. Items which cannot be adequately identified are marked as such.

In MIDI we are not able to distinguish between the enharmonic tones (C $\ddagger$  and D $\flat$ , for example) because both are represented by the same MIDI key number (61 in the octave of Middle C). Continued exposure to MIDI files caused us to realize that since a listener does not see a key signature or written accidentals, he or she must use other means to perceive stability and change in tonal contexts. Therefore, we reasoned, there should be some way to design an algorithm for the detection of key centers and key modulations in tonal music other than those which depend on the grammar of musical notation. In this regard, we have found the traditional concepts of harmonic theory as taught in our region to provide a foundation which is easily adapted to MIDI analysis.

## 9.2 Determining Tonality and Chordal Weight

#### 9.2.1 Scale Detection

Since not all files contain key signatures, our first procedure aims to establish the key of the piece. We assemble the first seven discrete pitches from the beginning of the composition and sort them into a test scale. This is evaluated for mode (major or minor). If it does not clearly conform to either mode, we add another pitch iteratively until a complete scale occurs among the sorted pitches.

We hold the structure of major- and minor-mode scales to be different insofar as the altered seventh degree occurs in minor only in the ascending harmonic-minor scale. We seek to identify one of five scales: (1) natural major, (2) harmonic major, (3) natural minor, (4) harmonic minor, and (5) melodic minor. (The harmonic major scale is a construct widely used in the study of Eastern European music. It differs from the natural major in having a flattened sixth degree.)



*Figure 9.1. Steps in the processes of note identification and chord evaluation.* (*Y*=*yes, N*=*no*).

| Event evaluation |   |   |  |  |  |
|------------------|---|---|--|--|--|
| Step             | Procedure   | Explanations, Assumptions   |  |  |  |
| 1                | Iterative input of the MIDI data: get all events within the first (next) beat-space.  | A beat-space is a window of time from one beat to the next.   |  |  |  |
| 2                | Read all pitches (whether coincident or succes-   | Efforts to assess the key of the  |  |  |  |
| 3                | If any pitch is found, skip to Step 6.  | notes have been presented and are   |  |  |  |
| 4                | If any pitch occurs on the next beat, go to Step 2.   | continued iteratively until a tonality can be inferred.   |  |  |  |
| 5                | If no pitch is found, end the program.  |   |  |  |  |
| Chord            | evaluation  |   |  |  |  |
| 6                | Sum together all durations of the same pitch (in various octave positions) in the current beat-space and evaluate the sums for every pitch.   | Higher values (within the<br>duration of one beat) identify<br>chord tones. Lower values are<br>melodic tones.                        |  |  |  |
| 7                | Using the pitches with the greatest cumulative<br>duration, determine whether the chord has<br>three members (triad) or four (seventh chord). | All chord types as used here are<br>composed of various combinations<br>of 3 and/or 4 half-steps. See Table<br>9.2.                   |  |  |  |
| 8                | If neither a triad nor a seventh chord can be identified, go to Step 10.  |   |  |  |  |
| 9                | Identify the root of the chord and apply a name from Table 9.1. Determine the root and label the chord (following Table 9.2). Go to Step 2.   |   |  |  |  |
| 10               | If, excluding the root, there are more than two pitches in the beat-space, go to Step 14.   |   |  |  |  |
| 11               | If the chordal intervals are a third and a fifth, go to Step 18.  |   |  |  |  |
| 12               | If there is a barline immediately following this beat-space, go to Step 2.  |   |  |  |  |
| 13               | Move the remaining pitches to the next beat-<br>space and go to Step 2.   |   |  |  |  |
| 14               | If a barline occurs in this beat-space, go to Step 2.   |   |  |  |  |
| 15               | Evaluate the pitches in the next beat-space (modulo 12).  |   |  |  |  |
| 16               | If a third or fifth occurs, go to Step 18.  |   |  |  |  |
| 17               | If no chord can be identified in this beat-space, output an x.  |   |  |  |  |
| 18               | If another third or fifth occurs in the next beat, go to Step 12.   |   |  |  |  |
| 19               | If a third is present without a fifth, output the sign of the chord with an asterisk (*) and go to Step 2.                                    | The chord is incomplete. There is<br>no way to be certain whether the<br>two tones present are prime and<br>third or third and fifth. |  |  |  |
| 20               | If a fifth is present without a third, output is x and go to Step 2.  | The chord is incomplete. There is<br>no way to distinguish between<br>major and minor triads.   |  |  |  |

**Table 9.1.** Explanation of steps shown in Figure 9.1.

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### 9.2.2 Chordal Evaluation

Next we compute chordal weights and their classes by quantifying the number, duration, and metrical location of all occurrences of each chord type. The many steps in The first beat of the bar is weighted more heavily than subsequent beats. Since in functional analysis it is not possible to determine the tonal function before detecting the tonality and root, functional issues are addressed in the last part of the processing. this process are noted in Figure 9.1 and described in Table 9.1.

## 9.2.3 Chord Labelling and Terminology

We defined chord types according to semitone patterns as shown in Table 9.2.

|         | Chordal<br>structure | Chord notated with<br>Middle C as root | Chord name                               | Chord<br>signifier |
|---------|----------------------|--|--|--------------------|
| Triads  | 4 - 3                |  | Major triad                              | +                  |
|         | 3 – 4                |  | Minor triad                              | _                  |
|         | 4 - 4                |  | Augmented triad                          | ++                 |
|         | 3 – 3                |  | Diminished triad                         | _                  |
| Tetrads | 4-3-3                |  | Major triad with minor seventh           | D7 (Maj–7)         |
|         | 3-3-3                |  | Diminished triad with diminished seventh | Dim7               |
|         | 3-3-4                |  | Diminished triad with minor seventh      | Dm7                |
|         | 4-3-4                |  | Major triad with major seventh           | Maj+7              |
|         | 3-4-3                |  | Minor triad with minor seventh           | Min–7              |

| Chordal<br>structure | Chord notated with<br>Middle C as root | Chord name                         | Chord<br>signifier |
|----------------------|--|------------------------------------|--------------------|
| 4-4-3                |  | Augmented triad with major seventh | Aug+7              |
| 3-4-4                |  | Minor triad with major seventh     | Min+7              |

*Table 9.2.* Chordal terminology as used here. Chordal structure is determined by patterns of major ("4") and minor ("3") thirds.

Our approach to chord labelling of triads is universal. Our labelling of tetrads ("sevenths") shares some features with many theoretical systems, but the idea of classifying all chords by their numbers and patterns of 3 and 4 half-step intervals (to the exclusion of seconds, for example) may be unfamiliar. [the tones shown for "D7", for example, would more conventionally be written A-C-Eb-Gb, but the interval sequence would remain 3-3-3.] Some of the chords could be rendered with different spellings without changing the MIDI key numbers (or the defining distances between them). The diminished seventh chord C-E=-G=-B==, for example, could occur as F<-A-C-E=. It would still be considered a diminished seventh, but with a root of F< (or, in Schoenberg's theory of extended tonality, as a ninth, with an absent root of D).

## 9.3 Implementation

#### 9.3.1 Data

To test our approach and implementation, we conducted a chordal analysis of 92 movements from works for piano of the eighteenth and nineteenth centuries. The MIDI data used comes from the Classical MIDI Archives (www.classicalarchives.com). The files each contain one movement from one of three composers — Wolfgang Amadeus Mozart (27 movements), Franz Schubert (33 movements), and Johannes Brahms (32 movements). Ideally, statistics should be culled from all the works of each composer, but at the present time there are few composers for whom all works have been encoded, unless they happen to have been composers who wrote very little music. While some results are trivial and others conform to expectations, some require discussion and merit further investigation. Does late Brahms differ from early Brahms more than early Brahms differs from Schubert, or Schubert from Mozart? In the past, such questions could be answered only subjectively or anecdotally. While in the current study we have not partitioned composers' careers to address such questions, we recognize that such studies are possible. Similarly, the piano works of Mozart and Schubert are overwhelmingly sonatas, while those of Brahms are mainly in other genres-ballades, intermezzi, and simply "keyboard" pieces.

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#### 9.3.2 Software for Chordal Identification

The current version of the software was written by Milan ðdimal (2004). It can be downloaded without cost from the website *http://www.sac.sk/files.php?d=10&l=D*. Select *dpan.zip*. The program labels MIDI data on the screen.

Inversions are identified by chordal structure as defined by the configuration of intervals of three or four semitones. We take the root of the chord to be the lowest tone of the uninverted chord. A chordal search, however, is started from the lowest tone in the current event (beat-space) of the score, i.e., the lowest MIDI key number. As the intervals formed by higher pitches are classified, the lower sounding tones may be reassigned to another octave (modulo 12) so that the theoretically correct root eventually occupies the lowest pitch position. Every inversion of a particular chord is placed in the same category as root-position and other inversions of that chord, according to the types defined in Table 9.2. The signs shown in the rightmost column appear under the appropriate events in the on-screen score. The overall procedure is clarified in Figures 9.2 and 9.3 [which have been reproduced in SCORE from the screen shots submitted].



Figure 9.2. Mozart piano sonata K. 333, Movement 1, Bars 3-4. Simple harmonic labeling.

In Figure 9.2, Bar 3, we find one triad (C, E=, G). The non-chord pitches on Beats 3 and 4 are of sixteenth-note rhythmical values, so they are too short to be considered here as members of adjacent chords. In Bar 4, on Beats 1-2, there are two pitches (B=, E=) not in a structure of thirds (F, A, C), so they are carried over to the next two beats, where there are four pitches (F, A, C, E=). The B= on Beats 1 and 4 is considered to be foreign. The label D7 (Beats 1-2) is assigned on the basis of its description in Table 9.2. Beat 3, containing the pitches F, A, C, repeats the label (+). The chordal pitches of Beat 4 appear to be C and E=, but is B= also part of the chord? If it is, then the chord could be a diminished triad with a minor seventh. If it is a passing tone (e.g., in relation to F-A-C), then the root is absent. [On absent roots, see Ch. 6.]

Common pitch-class values within one beat-space are combined. If, for example, four sixteenth notes at one pitch fall within one beat, their value is given as a quarter note.



*Figure 9.3.* Mozart's piano sonata in Bb Major, K. 333, Movement 1, Bars 11-12. CC = compound chord.

In Figure 9.3, Bar 11, there are only two pitches (B=, D) at the outset, so the tones are carried over to the next beat, where the addition of F completes the Bb triad. (A plus sign (+) is used to indicate the continuation of the information from one beat to the next.) On Beats 3 and 4 the numerical information is again insufficient to permit chordal identification. We give the indication CC (compound chord). The B= is carried over to the fourth beat, where it is still impossible to identify a chord. Chordal ambiguity continues until Bar 12, Beat 2, where a B=-major triad can be identified. Incomplete information arises again in Beats 3 and 4. The B= and D on Beat 3 are carried over to Beat 4, where they flow into a minor triad. Thus the minus sign (–) is allocated to Beats 3 and 4.

In the complex textures of piano music, melodic tones are excluded from vertical events if they do not belong to the current chord. For example, if any tone is neither three nor four semitones different from the current bass, or if its value is very short, it is excluded. Broken (arpeggiated) chords present a particular profile, since although all the members may be three or four semitones from the bass, each will enter at a different time.

The example from Brahms' Intermezzo Op. 119, No. 1 (Figure 9.4) exhibits broken chords (with chord members introduced in inverse order; Bars 1-3 and 9-11); chords divided between primary and secondary beats (Bars 4, 6); and passing tones and chords (Bars 5, 7-8). If in seeking a vertical structure of thirds the program fails to find a match, it moves to the next vertical structure. (See Figure 9.1. and Table 9.1). It is clear that any algorithmic approach to music of such tonal ambiguity makes chordal classification difficult. Bar 1 of this intermezzo can be "read" from left to right (triggering an idenfication of a B-minor triad at the third event) or from left to right (triggering an E-minor ninth-chord identification at the fifth event). Tonal ambiguity was part of Brahms' language, a fact that suggests the need for further research into algorithmic methods.

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Figure 9.4. Brahms ' Intermezzo in B Minor, Op 119, No. 1, Bars 1-11.

Other software used in our work includes a program called GUHA (General Unary Hypotheses Automaton), which has been developed over many years at the Czech Academy of Sciences in Prague by Petr Hájek (see, inter alia, Hájek et al. 2004). It is designed to test hypotheses by applying rules of association and implication to empirical data. Additionally, the software can generate its own hypotheses based on the properties of the data explored. Further information on GUHA is available at *www.cs.cas.cz*.

## 9.4 Findings

#### 9.4.1 Chordal Usage by Mode

We evaluate the eleven chordal types with respect to the mode (major or minor) of the fundamental key of each movement in the database. In the dataset, 59 movements are in major, 33 in minor. The distribution of chords in each mode is shown in Figure 9.5. It can be seen that while overall the proportions are similar, there is a significant difference in the usage rate of all triads, which is offset only slightly by the usage patterns for seventh chords.

#### 9.4.2 Chordal Usage by Composer

To create profiles of harmonic usage by individual composers (Mozart, Schubert, Brahms), we assess the frequency of occurrence of single-chord classes, two-chordclass sequences, and three-chord-class sequences. (The statistics for chord sequences are based only on chord transitions in which no miscellaneous notes intervene be tween beats.) A computation of the incidence of use of general chord classes is shown



*Figure 9.5.* Relative occurrence in major (grey) and minor (black) keys of the chordal types described in Table 9.2. Numbers have been normalized.

in Figures 9.6 (triads) and 9.7 (tetrads, or "seventh chords"). In the use of triads, we see that minor ones are used more prevalently by Mozart and Brahms than by Schubert. Schubert, however, shows a significantly greater use of major seventh chords, together with a reduced use of dominant sevenths. While sampling all the varieties of seventh chords, Brahms shows a proportionally high use of dominant sevenths. Mozart's use of minor sevenths is significantly greater than that of Brahms. Schubert's use of diminished minor sevenths is as slight as the use of augmented sevenths by all three composers.

Some of the results must be viewed against the changing preferences for mode. The Mozart movements are heavily weighted towards the major mode, while Schubert gives roughly even attention to major and minor. Brahms favors minor by almost 2:1. These differences refer only to key by movement, not to the more subtle issue of local tonalities that occur in the process of modulation within a movement.

In the breakdown of usage by individual chords, the variation within works by one composer can be greater than the difference between composers. Overall statistics are presented in Table 9.3. Transverse profiles of chord usage throughout the data collection are shown in Figure 9.8. In all cases Mozart and Schubert tend to cluster together, while Brahms makes entirely different choices.

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*Figure 9.6.* Distribution of triad classes in piano works by Mozart (left), Schubert (center), and Brahms (right). See legend below.

| Triads     | Tetrads             |
|------------|---------------------|
| 🗖 Major    | ■ Maj triad/min 7th |
| Minor      | ■ Dim triad/dim 7th |
| Augmented  | □ Dim triad/min 7th |
| Diminished | ⊡ Maj triad/maj 7th |
|            | ■ Min triad/min 7th |
|            | ■Aug triad/maj 7th  |
|            | ■ Min triad/maj 7th |



*Figure 9.7.* Distribution of tetrad (seventh-chord) classes in Mozart (left), Schubert (center), and Brahms (right). See legend above.

|         | Incidence |          |        | Chord when          |   | Chord         |
|---------|-----------|----------|--------|---------------------|---|---------------|
|         | Mozart    | Schubert | Brahms | Middle C as<br>root | Chord name  | signifier     |
| Triads  | 43.01%    | 43.89%   | 28.20% |                     | Major triad                                       | +             |
|         | 16.06%    | 19.80%   | 22.19% |                     | Minor triad                                       | -             |
|         | 2.74%     | 1.88%    | 5.51%  |                     | Augmented<br>triad                                | ++            |
|         | 4.3%      | 4.83%    | 8.01%  |                     | Diminished<br>triad                               | _             |
| Tetrads | 10.73%    | 15.74%   | 12.4%  |                     | Major triad with minor seventh                    | D7<br>(Maj–7) |
|         | 2.52%     | 3.15%    | 3.38%  |                     | Diminished<br>triad with<br>diminished<br>seventh | Dim7          |
|         | 0.88%     | 1.88%    | 5.51%  |                     | Diminished<br>triad with minor<br>seventh         | Dm7           |
|         | 7.90%     | 4.005%   | 7.33%  |                     | Major triad with major seventh                    | Maj+7         |
|         | 5.34%     | 3.46%    | 8.05%  |                     | Minor triad with minor seventh                    | Min–7         |
|         | 0.61%     | 0.58%    | 1.07%  |                     | Augmented<br>triad with major<br>seventh          | Aug+7         |
|         | 1.32%     | 0.77%    | 0.99%  |                     | Minor triad with major seventh                    | Min+7         |

*Table 9.3.* Differential incidence of chord types according to composer.



*Figure 9.8.* Incidence charts comparing the usage of particular chord types found in Mozart [left partition], Schubert [center partition], and Brahms [right partition]. The x-axis traverses the works in the composite database composer by composer. The y-axis shows the incidence of the event. The chord types are (a) major triads, (b) minor triads, (c) diminished triads, (d) dominant seventh tetrads [major triads with added minor seventh], (e) diminished/minor seventh tetrads [diminished triads with added minor seventh], and (f) minor seventh tetrads [minor triads with added minor seventh].

#### 9.4.3 Chordal-Transition Matrices

Our chordal matrices seek to profile the kinds of chords most likely to be used as the first or second in two-chord transitions and then as first, second, or third in threechord transitions. Statistical evaluations of two-chord transitions were computed only for works by Mozart and Schubert, which were relatively comparable in their harmonic traits. Mozart's music typically emphasizes seventh chords and the diminished triad, both as single chords and within chord sequences, while Schubert's music shows a still higher incidence of the dominant seventh chord and diminished/ minor seventh chords, both in isolation and in sequential chord pairs. See Table 9.4.

|                         | Mozart         | Schubert       |
|-------------------------|----------------|----------------|
|                         | Average number | Average number |
| Dominant 7th            | 10.73          | 15.74          |
| Dom. 7th–Dom. 7th       | 3.35           | 7.13           |
| Major 7th               | 7.90           | 4.01           |
| Dim. triad–Major 7th    | 0.69           | 0.11           |
| Diminished triad        | 8.88           | 4.83           |
| Major triad–Dim. 7th    | 0.04           | 0.06           |
| Diminished 7th          | 0.88           | 1.88           |
| Major 7th–Minor 7th     | 0.60           | 0.06           |
| Dim. triad–Major triad  | 1.84           | 0.83           |
| Major 7th–Major triad   | 2.85           | 1.12           |
| Dom. 7th–Dim. 7th       | 0.19           | 0.59           |
| Dim. triad–Dim. triad   | 4.03           | 1.87           |
| Minor triad–Dim. 7th    | 0.11           | 0.38           |
| Minor triad–Dim. triad  | 1.21           | 0.58           |
| Dim. 7th–Dom. 7th       | 0.05           | 0.32           |
| Minor triad-Minor triad | 8.26           | 11.81          |

*Table 9.4.* Averaged usage of the triads and tetrads singly [white background] and in chord pairs [grey background] in piano works by Mozart and Schubert.

In the music of Brahms it proved to be difficult to evaluate chord sequences for several reasons: (a) the total number of apparent chord types was very great, (b) many downbeat chords were highly complex (labelled CC for "complex chord" in the software), (c) there were many issues of rhythmic displacement (e.g., a downbeat without a chord), and consequently (d) few uninterrupted pairs could be identified.

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## 9.5 Conclusions and Future Work

We have succeeded in our aim of establishing profiles of chordal usage in the three piano repertories we examined. In a long series of statistical studies we have determined that among the important elements for determination of Mozart's harmonic procedures are the frequencies of diminished triads, minor seventh chords, minor/major seventh chords, and absence of the diminished/minor seventh. The music of Brahms is more difficult to evaluate by our methods than that of Mozart or Schubert partly because his usage of major triads is lower overall but mostly because he concentrates more on the outer edges of tonality than do his predecessors. Brahms' harmonic usage can be defined by the quantity of diminished/minor sevenths and the suppression of major triads, dominant sevenths, diminished triads, and diminished seventh chords.

Our future work will be oriented towards exploring unexpected features of the harmonic treatment in these repertories and also adapting our software to accommodate the idiosyncracies of individual composers. We find, for example, that unusual metrical features of Brahms' music produce results which are difficult to anticipate in a template designed for general use. Many other stylistic features cause us to consider further elaboration of the procedures. In terms of specific works, however, it is very difficult in certain instances to evaluate the dynamic aspects of harmonic change.

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